Abstract: This paper aims at the control of variable frequency air-conditioning system that has characteristic as large inertia and pure lag. The neural network PID control in the variable-frequency air-conditioning system is introduced and simulated. In the learning algorithm of neural network PID controller, the output of system is needed to tune the weight of neural network while it is difficult to obtain. So the output of system is predicted through the algorithm of nonlinear that adopts the neural network configure. Through simulation and optimization, it is found that the neural network PID control has the capability of self study and self adaptation. However, the neural network PID control system sometimes has the static error. To eliminate the static error, the hybrid control of neural network PID and conventional PID is applied to the variable-frequency air-conditioning system. The hybrid control is simulated to compare the performance of changed parameters of model. The simulation finds that the hybrid control of neural network PID and PID has both the advantages of neural network and PID, such as self-studying and self-adapting and obtain faster response and better performance.

Keywords: neural network; PID control; prediction; variable-frequency; air-conditioning system

I. INTRODUCTION

Currently it is a focus study to apply the intelligent control to the air-conditioning system, which is helpful to not only satisfy the comfort of human, but also save energy. Conventional PID controller sometimes doesn’t satisfy the control purpose to the object, which is large inertia, delay and non-linear characteristic and uncertain disturbance factor. For overcoming the failure, some people begin to grope for the intelligent control and apply to the control of variable-frequency air-conditioning system.

The optimization of PID parameters is one of ways and methods to improve the PID control in the heating, ventilation and air-conditioning (HVAC) system. Many studies are involved in the parameter optimization, for example in [1] to [3], while the optimization methods are different. In [1] an adaptive learning algorithm based on genetic algorithms (GA) for automatic tuning of PID controllers in HVAC systems to achieve optimal performance is presented. The neuron adaptive PID controller is applied in a single-zone HVAC system and the simulation results shows that the output of neuron PID control first enter stable belt and the anti-interference to white noise is stronger than PID control in [2]. X. Qi aims at the variable air volume (VAV) air-conditioning system and designs a fuzzy PID controller in [3]. From the simulation results it is seen that the fuzzy PID controller has fast response, small overshoot, high accuracy, strong robust and self-setting in line when the parameter changes.

The studies of variable-frequency air-conditioning system mainly apply the fuzzy control algorithm as in [4] to [6]. In [4] the characteristics such as ITAE, coefficient of performance (COP) and the control precision have been compared in fuzzy control and traditional PI control for inverter air conditioner. Optimization of fuzzy control rules and membership function has been done based on genetic algorithm (GA) in [5]. In [7] the fuzzy self-adaptation control was introduced and used in inverter air-conditioning that has the quality of self-adaptation for the changed parameter of controlled objects. In [8] the controller combined CMAC neural network and fuzzy logic control and was simulated in the inverter air conditioner.

Additionally the neural network technology has been widely applied in the HVAC field to control, energy management, prediction, fault diagnosis, identification and optimization, etc [9]. Some researchers paid attention to the neural networks control for variable-frequency air-conditioning system. In [10] the algorithm using BP neural networks was optimized in frequency inverter air conditioner. Ref. [11] develops a neural PID control algorithm module to simulate a variable-frequency air-conditioning system, which shows that the neural PID control strategy is more robust than conventional PID control. Reference [12] simulated three controllers including PID controller, fuzzy controller and neural network controller in variable frequency central air-conditioning system.

This paper aims at the control of variable-frequency air-conditioning system and introduce the design of neural network PID controller. Through simulation and optimization, it is found that the neural network PID control has the capability of self-study and self-adaptation. However, the neural network PID control system sometimes has the static error. To eliminate the static error, the hybrid control of neural network PID and conventional PID is advanced in variable-frequency air-conditioning system, which has both the advantages of neural network and PID.

II. NETWORK PID CONTROLLER

A. Configuration of Neural Network PID Control System

The neural network PID control is the combine of the conventional PID control and neural network, which mingle the excellence of PID and neural network. The structure of the...
neural network PID control system consists of the neural network PID controller and the nonlinear prediction model (NNM) as shown in Fig. 1.

![Figure 1. Neural network PID control system](image)

In Fig. 1 r is the setting input, \( \Theta(5) \) is the controlled object, \( y \) is the actual output, \( u \) is the control variable, \( \hat{y} \) is the prediction of \( y \). NN stands for the neural network, NNM stands for the neural network model to predict the \( \hat{y} \) and \( F \) stands for the learning algorithm.

The NN in Fig. 1 bases on the state of system and adjust the PID controller's parameters to obtain the optimization of some performance index. When the NN is adjusted in a learning algorithm, it is possible that the prediction of output or the variation is used to calculate the control value or tune the weights of neural network as in [13] and [14]. But the prediction output of system is so difficult to obtain that the usual method is to model the controller object and to predict the output. Thus the actual output can be instead of the prediction output to calculate the control value or tune the weights of neural network. Here the NNM in Fig. 1 is the prediction model of controlled object and to predict the output \( \hat{y} \).

### B. Neural Network PID Controller

The control system with neural network PID controller is constructed as in Fig. 1. Now, let the error be denoted by \( e(k) = r(k) - y(k) \) where \( r(k) \) is a desired signal and \( y(k) \) is an actual output of the plant. In the discrete-time control system, the PID algorithm can be given by

\[
u(k) = u(k-1) + K_p(e(k) - e(k-1)) + \]
\[
K_i e(k) + K_d (e(k) - 2e(k-1) + e(k-2)).
\]

(1)

where \( K_p, K_i, K_d \) are respectively the proportional, integral and derivative gains of the PID controller which should be tuned and optimized.

Equation (1) can be recomposed as follows:

\[
u(k) = f_u(u(k-1), K_p, K_i, K_d, e(k), e(k-1), e(k-2)).
\]

(2)

where \( f_u \) is a nonlinear function that is concerned with \( K_p, K_i, \) and \( K_d \). The best control rule can be discovered by the training and learning of back propagation (BP) neural network.

The NN module in Fig. 1 can be used to adjust the gains of PID controller adaptively by using the BP method with measurement data of \( u(k), y(k), \) and \( r(k) \). The BP network is a multilayered network which consists of an input layer, an output layer, and several hidden layers of nonlinear processing elements. In this paper, the three layers BP NN is used as Fig. 2, which has M input neurons, Q hidden neurons and three output neurons. The input neurons can be the state of system, for example the input or output of different times, which should be normalized as [15] if the network needs. The output of BP NN, respectively, is the three tuned parameters of PID controller. Because the three parameters \( K_p, K_i, \) and \( K_d \) can't be negative, the activation function of output layer's neurons is Sigmoid function that is not negative and the activation function of hidden layer's neurons is Sigmoid function that is symmetry of positive and negative.

In Fig. 2 the output of input layer's neurons of BP neural network can be expressed as follows:

\[
o_j^{(0)} = x_{w,j} = e(k-j), j = 0,1,...,M-1,
\]
\[
o_j^{(0)} = 1.
\]

(3)

where \( o_j^{(0)} \) is the output of \( j \)-th neuron in input layer, the number of input layer's neurons M lies on the complex degree of the control system. In the equations, the superscript symbols (1), (2) and (3) are, respectively, the input layer, hidden layer and output layer.

The input and output of hidden layer can be expressed:

\[
et_{ij}^{(2)}(k) = \sum_{i=0}^{M} w_{ij}^{(2)} o_i^{(1)}(k),
\]
\[
o_i^{(1)}(k) = f_{i}(\text{net}_{i}^{(2)}(k)), i = 0,1,...,Q-1,
\]
\[
o_0^{(1)} = 1.
\]

(4)

where \( \text{net}_{i}^{(2)} \) is the input of \( i \)-th neuron in hidden layer, \( w_{ij}^{(2)} \) is the weights of hidden layer, \( o_i^{(1)} \) is the valve value; \( f_i \) is the activation function that is \( f_i = \tanh(v) \)

The input and output of output layer can be expressed:

\[
et_{ij}^{(3)}(k) = \sum_{i=0}^{Q} w_{ij}^{(3)} o_i^{(2)}(k),
\]
\[
o_j^{(3)}(k) = g(\text{net}_{i}^{(3)}(k)), l = 0,1,2,
\]
\[
K_p(k) = o_1^{(3)}(k),
\]
\[
K_i(k) = o_2^{(3)}(k),
\]
\[
K_d(k) = o_3^{(3)}(k).
\]

(5)
where \( w_n^{(o)} \) is the weights of output layer, \( v_n^{(o)} \) is the valve value, \( v_n^{(o)} = \theta \) \( g[] \) is the activation function and \( g[]-[1+tanh(x)]/2 \).

By using the BP algorithm based on gradient method, to minimize the performance index function \( J \) which can be expressed as following:

\[
J = \frac{1}{2} [r(k+1) - y(k+1)]^2 = \frac{1}{2} e^2(k+1). \tag{6}
\]

where \( J \) is to modify the weights by the fastest descend mean, which is searched and tuned toward the negative gradient and added on a inertia coefficient to make faster stringency, and:

\[
\Delta w_n^{(3)}(k+1) = -m \frac{\partial J}{\partial w_n^{(3)}(k)} + \alpha \Delta w_n^{(3)}(k). \tag{7}
\]

where \( m \) stands for the velocity of learning, \( \alpha \) is smoothing coefficient.

\[
\frac{\partial J}{\partial x_n^{(3)}} = \frac{\partial J}{\partial y(k+1)} \frac{\partial y(k+1)}{\partial u(k)} \frac{\partial u(k)}{\partial o_n^{(3)}(k)} \frac{\partial o_n^{(3)}(k)}{\partial \tilde{u}(k)} - \alpha \frac{\partial J}{\partial x_n^{(3)}}. \tag{8}
\]

In (8) \( \frac{\partial y(k+1)}{\partial u(k)} \) is known so that it can be replaced by \( \frac{\partial y(k+1)}{\partial u(k)} \), which can be calculated in the nonlinear model or least-squares method. Here \( \frac{\partial y(k+1)}{\partial u(k)} \) is predicted in neural network.

In (1) the difference of \( u(k) \) to \( K_p, K_i, K_d \) can be expressed as following:

\[
\begin{align*}
\frac{\partial u(k)}{\partial w_0^{(3)}}(k) &= e(k) - e(k-1), \\
\frac{\partial u(k)}{\partial a_0^{(3)}}(k) &= e(k), \\
\frac{\partial u(k)}{\partial a_2^{(3)}}(k) &= e(k) - 2e(k-1) + e(k-2).
\end{align*} \tag{9}
\]

So, the weights of output layer in BP neural network are updated as following:

\[
\begin{align*}
\Delta w_n^{(3)}(k+1) &= -m \delta_n^{(3)}(k) o_n^{(3)}(k) + \alpha \Delta w_n^{(3)}(k), \\
\delta_n^{(3)}(k) &= e(k+1) \cdot \frac{\partial y(k+1)}{\partial u(k)} \cdot \frac{\partial u(k)}{\partial o_n^{(3)}(k)} \cdot g'[\tilde{u}(k)], \\
i &= 0,1,2.
\end{align*} \tag{10}
\]

According as the mean like above, the weights of hidden layers are updated as following:

\[
\begin{align*}
\Delta w_n^{(2)}(k+1) &= m \delta_n^{(2)}(k) o_n^{(2)}(k) + \alpha \Delta w_n^{(2)}(k), \\
\delta_n^{(2)}(k) &= e(k+1) \cdot \frac{\partial y(k+1)}{\partial u(k)} \cdot \frac{\partial u(k)}{\partial o_n^{(2)}(k)} \cdot g'[\tilde{u}(k)], \\
i &= 0,1,2.
\end{align*} \tag{11}
\]

where \( \delta_n^{(2)}(k) \) is the weights of hidden layer, \( w_n^{(2)} \) is the valve value and \( \theta_{\delta_n^{(2)}} = \theta \), \( g[] \) is the activation function.

And the output of output layer can be expressed as following:

\[
\begin{align*}
\Delta w_n^{(1)}(k+1) &= m \delta_n^{(1)}(k) o_n^{(1)}(k) + \alpha \Delta w_n^{(1)}(k), \\
\delta_n^{(1)}(k) &= e(k+1) \cdot \frac{\partial y(k+1)}{\partial u(k)} \cdot \frac{\partial u(k)}{\partial o_n^{(1)}(k)} \cdot g'[\tilde{u}(k)], \\
i &= 0,1,2.
\end{align*} \tag{12}
\]

C. Nonlinear Prediction

In the variable-frequency air-conditioning system, the controlled object is supposed to a nonlinear system that is single input and single output system as following:

\[
\begin{align*}
y(k) &= f[y(k-1),y(k-2),...,y(k-n_y), \\
u(k-1),u(k-2),...,u(k-n_u)]
\end{align*} \tag{13}
\]

where \( y(k) \) and \( u(k) \) are the output and input on the \( k \) time, \( n_y \) and \( n_u \) are the order of \( y \) and \( u \), \( f[] \) is the nonlinear function.

To calculate the prediction of \( \tilde{y}(k+1) \) or \( \frac{\partial \tilde{y}(k+1)}{\partial u(k)} \), there is a three layers BP neural network model (NNM in Fig. 1) as the prediction model which has \( n_y+1 \) input neurons, \( Q \) hidden neurons and one output neuron. For the sake of prediction of nonlinear system easily, the activation function of output layer is linear function and the activation function of hidden layer's neurons is still Sigmoid function.

The prediction calculation of BP neural network model: let the input and output of the plant \( \{y(k)\} \) and \( \{u(k)\} \) be the neural network model's input, and the input layer:

\[
\begin{align*}
o_j^{(0)}(k) &= \begin{cases} y(k-j) & 0 \leq j \leq n_y-1 \\
u(k-j+n_u) & n_y \leq j \leq n_y+n_u-1 \end{cases}, \\
o_{n_y+n_u}^{(0)}(k) &= 1
\end{align*} \tag{14}
\]

The input and output of hidden layer can be expressed:

\[
\begin{align*}
\tilde{u}(k) &= \sum_{i=0}^{n_y+n_u} w_{y_i}^{(0)} o_j^{(0)}(k), \\
o_i^{(2)}(k) &= f[\tilde{u}^{(2)}(k)], \\
o_i^{(2)}(k) &= 1, \\
i &= 0,1,...,Q-1.
\end{align*} \tag{15}
\]

where \( w_{y_i}^{(0)} \) is the weights of hidden layer, \( w_{y_i}^{(2)} \) is the valve value and \( \theta_{o_i^{(2)}} = \theta \), \( f[] \) is the activation function and \( f[] = tanh(x) \).

And the output of output layer can be expressed as following:
\[ y(k+1) = \sum_{i=0}^{Q} w_i^{(2)} o_i^{(2)}(k). \]  
(16)

where \( w_i^{(2)} \) is the weights of output layer, \( w_i^{(1)} \) is the valve value and \( w_i^{(0)} = \theta_i \), and the output neuron is linear neuron.

The backward learning of BP neural network model: use the learning algorithm of BP to modify the weights and valve value and make the target function \( J_r \) minimum:

\[ J_r = \frac{1}{2} \sum_{k=0}^{P} (y(k+1) - y(k))^2. \]  
(17)

and the weights is modified as following:

\[ \Delta w_i^{(2)}(k) = m[y(k+1) - \hat{y}(k+1)]o_i^{(2)}(k) + \alpha \Delta w_i^{(2)}(k), \]
\[ \Delta w_i^{(1)}(k) = m[y(k+1) - \hat{y}(k+1)]f[\text{net}_i^{(2)}(k)]w_i^{(2)}(k) + \alpha \Delta w_i^{(1)}(k), \]
\[ i = 0,1,...,Q, \]
\[ j = 0,1,...,n_q + n_e, \]
where \( m \) is the velocity of learning, \( \alpha \) is smoothing coefficient, and both is in the \((0,1)\)

The derivative of activation function can be expressed:

\[ f'(x) = (1 - f^2(x))/2. \]  
(19)

So \( \frac{\partial \hat{y}(k+1)}{\partial u(k)} \) can be expressed as following:

\[ \frac{\partial \hat{y}(k+1)}{\partial u(k)} = \sum_{i=0}^{Q} \frac{\partial y(k+1)}{\partial o_i^{(2)}(k)} \frac{\partial o_i^{(2)}(k)}{\partial \text{net}_i^{(2)}(k)} \frac{\partial \text{net}_i^{(2)}(k)}{\partial u(k)} \]
\[ = \sum_{i=0}^{Q} (\alpha f'[\text{net}_i^{(2)}(k)]w_i^{(2)}(k)). \]  
(20)

D. Algorithm Procedure

Through the above analysis of neural network PID control, the calculation procedure of BP neural network PID control that adopts the nonlinear prediction model can be summarized as following:

a) Select the BP neural network configure: define the input neurons M and hidden neurons Q, initialize the weights of each layer \( a_0^{(i)}(0) \) and \( a_0^{(i)}(0) \) and select the learning velocity \( \alpha \) and smoothing coefficient \( \alpha \), moreover \( k=1 \).

b) Sample \( r(k) \) and \( y(k) \) and calculate \( e(k) = z(k)-r(k)-y(k) \).

c) Normalize \( r(k), y(k), u(k-1) \) and \( e(i) \) \( (i=k,k-1, ...,k-p) \), which these parameters are the inputs of NN.

d) Based on (3) to (5), the forward calculation of BP neural network is carried out and the tuned PID parameters \( K_p(k), K_i(k) \) and \( K_d(k) \) are output.

e) Based on (1), \( u(k) \) is calculated and it is used to control and other calculation.

f) Based on (14) to (16), the forward calculation of neural network of NNM is carried out and the prediction \( \hat{y}(k+1) \) is output.

g) Based on (18), the weights of hidden layer and output layer are modified.

h) Based on (20), \( \frac{\partial \hat{y}(k+1)}{\partial u(k)} \) is calculated.

i) Based on (10), the weights of output layer \( a_0^{(2)}(k) \) are modified.

ej) Based on (11), the weights of hidden layer \( a_0^{(2)}(k) \) are modified.

k) Let \( k=k+1 \) and return step b).

III. SIMULATION

A. Simulation Model

The system consists of variable-frequency air-conditioning and the room, which is object of simulation. Based on [16], the transfer function as follows shows the relation of the room temperature and the compressor frequency, which is first-order inertial segment with delay.

\[ H(s) = \frac{k}{1 + Ts}. \]  
(21)

In the simulation model, the amplification coefficient \( k = 0.3 \) °C/Hz, the characteristic time \( T = 1800s \) and the delay time \( r = 120s \). These setting reflect the characteristics of pure lag and inertia of air-conditioning system. Based on the transfer function, the neural network PID control and traditional PID control were simulated.

B. Simulation

The initial temperature of room is 0°C and the setting temperature is 20°C. The upper limit of compressor frequency is 110Hz. The sample time is 30 seconds as in [17].

Through the simulation and optimization, the traditional PID parameters are \( K_p = 50, K_i = 0.12, K_d = 60 \), which also are the corresponding gains of neural network outputs. The neural network PID in the simulation is three input neurons, eight hidden neurons and three output neurons, the velocity of learning \( \eta = 0.3 \), the flatness coefficient \( \alpha = 0.1 \), and the initial weights is the random number in the extent [-0.5, 0.5].

Figure 3 displays the simulation results of neural network PID and PID. It is known that the PID controller can obtain better performance of system through optimization and multiple simulations. The neural network PID obtains smooth transition, but the static error exists.

To compare the robust, the model parameters are changed. Figure 4 compares the performance when the delay time increases 50% and \( r = 180s \). It is found that at last the PID control oscillates in constant amplitude, while the neural network PID control stabilizes smoothly, at the same time the static error still exists.
Figure 5 compares the performance when the amplification coefficient $k = 0.36 \degree C/Hz$. The result is similar to Fig. 4.

Figure 6 compares the performance when the white noise is added into the control system. It is found that the resistance to noise of neural network PID control is better than PID control, which is favorable to the compressor's smooth run.

From the above simulation, it can be found that as long as the parameters of PID control are optimized, conventional PID control obtains better performance. The neural network PID control can also obtain smooth transition, but the system sometimes has the static error.

The neural network PID control has the capability of self-study and self-adaptation. When the parameters of model change, the neural network PID control still obtains smooth transition. The robust of neural network PID is stronger than PID control.

C. Hybrid Control

To eliminate the static error of neural network PID, the hybrid control system is constructed in neural network PID and conventional PID. When the error of actual output and the setting is less than the setting threshold quality, the control system switches the PID control. When the error is greater than the setting threshold quality, the control system switches the neural network PID control.

Through the simulation and comparison, the threshold quantity is chosen to $0.5^\circ C$. Figure 7 compares neural network PID control and PID control when the threshold quantity is chosen to $0.5^\circ C$. From the results, it is seen that the hybrid control and PID control both obtain better performance and the hybrid control eliminates the static error of neural network PID control only, but the overshoot of hybrid control is less than the PID control and the transient time is shorter than the PID control.

Figure 8 compares the robust when the delay time increases 50% and $\tau = 180s$. It is found that the outputs of PID control and hybrid control both oscillate, but the oscillation amplitude of hybrid control is relatively less to the PID control and the hybrid control can reach the stable state.
IV. Conclusion

As long as the parameters of PID control, proportion gain, integral time, differential time and sampling period, are optimized and tuned, conventional PID control obtains better performance and higher control precision in variable-frequency air-conditioning system. However, the robust of PID is weak when the parameters of model change.

The neural network PID control, which is a method for adaptively adjusting the PID gains using a BP neural network, can be adopted the variable-frequency air-conditioning system. The neural network PID control has the capability of self-study and self-adaptation. However, the neural network PID control system sometimes has the static error.

The hybrid control of neural network PID and conventional PID is applied to the variable-frequency air-conditioning system to eliminate the static error. The hybrid control of neural network PID and PID has both the advantages of neural network and PID, such as self-studying and self-adapting and obtain faster response and better performance.

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