Complex networks-based energy-efficient evolution model for wireless sensor networks

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\textbf{A R T I C L E   I N F O}

Article history:
Accepted 21 July 2008

Communicated by Prof. Ji-Huan He

\textbf{A B S T R A C T}

Based on complex networks theory, we present two self-organized energy-efficient models for wireless sensor networks in this paper. The first model constructs the wireless sensor networks according to the connectivity and remaining energy of each sensor node, thus it can produce scale-free networks which have a performance of random error tolerance. In the second model, we not only consider the remaining energy, but also introduce the constraint of links to each node. This model can make the energy consumption of the whole network more balanced. Finally, we present the numerical experiments of the two models.

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1. Introduction

Wireless sensor networks (WSNs) are usually made up of hundreds even thousands of distributed sensor nodes organized in ad-hoc paradigm to monitor physical phenomenon. WSNs can cover a wide range of applications domains, since they can be easily deployed and self organized. The applications of WSNs range from important societal issues such as environmental and habitat monitoring, traffic control, emergency scenarios, and health care, to economical issues such as production control and structure monitoring [1]. As sensor nodes are battery operated and many applications of WSNs require thousands of sensor nodes which will be deployed in remote locations, the battery replacement is impractical. Therefore, how to prolong the lifetime of the network is a very important concern in the study of wireless sensor networks.

Recently, there has been considerable interest in the structure and dynamics of large complex networks across many fields of science [2]. Undoubtedly, many systems in nature can be described by the models of complex networks, which are the structures consisting of nodes connected by links, such as social networks [3,4], scientific collaboration networks [5], the World Wide Web [6,7], and metabolic networks [8]. In fact, many research has been done to give a new method to develop strategies against terrorism [9] and the topology of the Italian airport network [10]. Moreover, Barabasi and Albert [11,12] proposed a scale-free model, by which a complex network can be generated with a power-law degree distribution. It is shown that scale-free networks are robust against random removal or failures of nodes, but are fragile when the most connected nodes are targeted [13–15].

Many energy-aware and fault-tolerant topology control algorithms for wireless sensor networks have been presented in recent years. In Ref. [16], authors proposed an energy-efficient routing algorithm for gathering correlated data in sensor networks. Authors in Ref. [17] proposed an approach to construct $k$-connected network for clustering sensors deployed in hostile environments. Authors in Ref. [18] presented a mechanism to deduce fault-tolerant communication topology among the cluster heads. Several control algorithms considered in Ref. [19] are to maintain network connectivity while improving...
network performance. Authors in Refs. [20,21] have much related work about energy efficiency and fault tolerance. However, almost all the existing studies have not studied the performance of energy efficiency and fault tolerance in WSNs from the views of the network evolution and degree distribution.

In this paper, we propose two evolving algorithms for wireless sensor networks based on complex networks theory. The first model is to deduce energy-aware communication topology, and this model can produce scale-free networks which have a performance of random error tolerance. In order to prevent high degree nodes consuming their energy quickly in data transmission, we limit the number of communication links of each node in the second model.

The remainder of this paper is organized as follows. In Section 2, we propose an algorithm of energy-aware evolution model. In Section 3, we propose an algorithm of energy-balanced evolution model. In the Section 4, we give the numerical experiments to present the features of the networks generated by the proposed algorithms. Finally, Section 5 gives the conclusion of this paper.

2. Energy-aware evolution model of WSNs

Although being one kind of complex networks, WSNs have their special features. Therefore, we must consider the factors such as the constraint of node transmission radius, energy efficiency, and their performance of fault tolerance. The local-area and fitness models in complex networks theory give us a new view for the study of the evolution algorithm while considering the factors of the constraint of the transmission radius and the remaining energy of each node. In this section, we present the first algorithm in energy-aware evolution model (EAEM).

In WSNs, the energy of each node is consumed in different use and the nodes spend their most energy in data transmission after the networks have been organized already, so we assume that the remaining energy of the nodes is various and fixed in the process of our evolution algorithms. And in wireless sensor networks, because of the constraint of node transmission radius, each node in the network can only communicate with those nodes locating in its transmission range which named local-area connections. As a result, the number of neighbors per node should be bounded by a (small) constant here.

For clarity, we list some important parameters used to characterize the wireless sensor networks in Table 1.

The generation of a network is as follows:

1. Growth: starting with a small number \(n_0\) of nodes, at every time step, we add a new node with \(m\) (\(m < n_0\)) edges (that will be connected to the nodes already present in the network).

2. Preferential attachment: When a new node comes into the network, it will choose some nodes in its local-area to connect. We assume that the probability \(P\) of a new node will be connected to node \(i\), its remaining energy \(E\) of that node. In this paper, we define a function \(f(E)\) to present the relationship between the remaining energy of a node and its ability to be linked. The more energy a node has, the more ability it will have of being connected to the new coming nodes. Therefore, \(f(E)\) must be an increasing function here and the form may be such as \(E, E^2, \sqrt{E}, \ln(E)\) and so on. And the form of \(\prod\) is

\[
\prod_i = \frac{f(E)k_i}{\sum_{j \in \text{local-area}} f(E_j)k_j}.
\]

In complex networks, the degree distribution is a very important and useful factor to observe the features of networks. And the degree distribution of networks created by using this algorithm can be examined based on the continuum theory, a method developed by Barabasi and Albert, in which network growth is treated as a continuous process to allow simplification of the model using calculus [22]. Such an approximation should match closely with discrete network growth, provided that we consider networks of sufficiently large scale, i.e., networks that undergo a large number of time steps. During each unit of time, \(m\) new edges are formed. So we get

\[
\frac{\partial(k_i)}{\partial(t)} \approx m \prod_i = \frac{f(E)k_i}{\sum_{j \in \text{local-area}} f(E_j)k_j}.
\]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Definition in WSNs</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n_0)</td>
<td>Number of nodes before beginning the evolution of network</td>
</tr>
<tr>
<td>(m)</td>
<td>Number of new edges connected to a new node at every time step</td>
</tr>
<tr>
<td>(k_i)</td>
<td>Number of links connected to node (i)</td>
</tr>
<tr>
<td>(E)</td>
<td>Remaining energy of a node</td>
</tr>
<tr>
<td>(\prod)</td>
<td>Probability of a newly coming node that will be connected to node (i)</td>
</tr>
<tr>
<td>(L)</td>
<td>Number of nodes in every new comer’s local-area</td>
</tr>
<tr>
<td>(\rho)</td>
<td>Distribution of the remaining energy of all the nodes in the network</td>
</tr>
<tr>
<td>(t_i)</td>
<td>Time of node (i) newly introduced into the network</td>
</tr>
</tbody>
</table>
In the local-area of each node, we have
\[ \sum_{f \in \text{local-area}} f(E)k_i = L\mathbb{E}(k), \]
where \( L \) is the fixed number of nodes in the new comer’s local-area, \( \mathbb{E} \) is the expected value of \( f(E) \), and \( \langle k \rangle \) is the average degree of the network. In a large scale network, the average degree can be calculated as
\[ \langle k \rangle = \frac{2(mt + e_0)}{m_0 + t} \approx 2m, \]
where \( e_0 \) and \( m_0 \) represent the number of links and nodes at beginning, respectively. And they are very small.

Substitute Eqs. (3) and (4) into Eq. (2), we have
\[ \frac{\partial f(k_i)}{\partial (t)} = \frac{f(E)k_i}{2L\mathbb{E}}. \]
Then we arrive
\[ \frac{\partial f(k_i)}{k_i} = \frac{f(E)}{2L\mathbb{E}} \, dt, \]
As \( f(E) \) is an increasing function, we set \( f(E) = E \) as an example, then
\[ k_i = e^{\frac{E}{m} + c}, \]
and lately we will show that the form of \( f(E) \) indeed do not influence the final result of the degree distribution feature.

Since \( k_i(t_i) = m \), then \( C = \ln m - \frac{E}{2E} t_i \), thus we have
\[ k_i = e^{\frac{E}{m}(-t_i)} \cdot m. \]

The probability that a node has a connectivity \( k_i(t) \) smaller than \( k \) is
\[ P(k_i(t) < k) = P\left( \frac{E}{2L\mathbb{E}} (t - t_i) < \ln(k/m) \right) = P\left( t - \frac{2L\mathbb{E}}{E} \ln(k/m) < t_i \right). \]
Assume that we add the node to the network at equal time intervals, the probability density at the time \( t_i \) is \( P_i(t_i) = \frac{1}{m_0 + t} \), therefore, we get
\[ P(k_i(t) < k) = 1 - P(t_i < t - \frac{2L\mathbb{E}}{E} \ln(k/m)) = 1 - \frac{t - \frac{2L\mathbb{E}}{E} \ln(k/m)}{m_0 + t}. \]
The probability density function of the degree of a node with remaining energy \( E \) is
\[ P(k_E) = \frac{\partial P(k_i(t) < k)}{\partial k} = \frac{1}{m_0 + t} \cdot \frac{2L\mathbb{E} m}{E \cdot k}. \]
To obtain the overall probability density function, we take a weighted average of these functions with the weights being the probabilities of having remaining energy \( E \). Moreover, \( k \) is the continuous random variable for node degree. In other words,
\[ P(k) = \int_{E_{\text{min}}}^{E_{\text{max}}} \rho P(k_E) \, dE = \int_{E_{\text{min}}}^{E_{\text{max}}} \rho \frac{1}{m_0 + t} \cdot \frac{2L\mathbb{E} m}{E \cdot k} \, dE, \]
where \( \rho \) is the distribution of \( E \). \( E_{\text{max}} \) and \( E_{\text{min}} \) are the bounds of remaining energy values, obviously \( P(k) \propto \lambda k^{-1} \), where \( \lambda = \int_{E_{\text{max}}}^{E_{\text{min}}} \rho \frac{1}{m_0 + t} \cdot \frac{2L\mathbb{E} m}{E} \, dE \), thus we can find that \( \lambda \) is a constant in a given network.

Therefore, the distribution has a power-law form with degree exponent \( \gamma = 1 \) despite of the distribution of remaining energy in every network. Thus, we can organize WSNs with scale-free feature in this energy-aware way. This algorithm can not only make the network evolution in an energy-efficient way, but also improve the network reliance against random errors, which is an inherent advantage of most scale-free networks.

3. Energy-balanced evolution model

In the above EAEM model, we can get the scale-free network structure in an energy-efficient way. However, in wireless sensor networks there are some sensor nodes with more energy which may connect too many nodes and run out of power quickly during working time after the self-organization of the networks. Moreover, these nodes connected with many links may become bottleneck of the network. To prevent nodes connecting with too many nodes, in this section, we propose another algorithm named energy-balanced evolution model of wireless sensor networks (EBEM). In this model we suppose that each node can connect no more than \( k_{(\text{max})} \) nodes (its maximum degree), and the number of \( k_{(\text{max})} \) also depends on its remaining energy \( E \). We assume that a node with the maximum energy \( E_{\text{max}} \) can connect to no more than \( k_{\text{max}} \) nodes. Therefore, we can easily get \( k_{(\text{max})} = k_{\text{max}} \frac{E}{E_{\text{max}}}. \)
Compared with the EAEM algorithm, we introduce the maximum degree limitation to the new model, and we get EBEM similarly as follows:

1. **Growth:** Starting with a small number \((m_0)\) of nodes, at every time step, we add a new node with \(m(m < m_0)\) edges (that will be connected to the nodes already present in the network).

2. **Preferential attachment:** When choosing the nodes to which the new node connects, we assume that the probability \(\prod_i\) of a new node being connected to node \(i\) depends on the connectivity \(k_i\), remaining energy \(E\) of that node. Node \(i\) will be connected to no more than \(k_{i(\text{max})}\) nodes, so

   \[
   \prod_i = \frac{f(E, k_i)k_i}{\sum_{j\text{-local-area}} f(E, k_j)k_j},
   \]

   where we define \(f(E, k_i) = E\left(1 - \frac{k_i}{k_{i(\text{max})}}\right)\), and \(k_{i(\text{max})} = k_{\text{max}}E_{\text{max}} 1 - \frac{k_i}{k_{i(\text{max})}}\) indicates that when the more closer \(k_i\) to \(k_{i(\text{max})}\), the less probably the node will be connected to the newly coming nodes. When \(k_i\) is closer to it’s maximum degree \(k_{i(\text{max})}\), the node \(i\) will not connect the newly coming nodes any more.

To get the degree distribution \(P(k)\), similarly we can use the method as in the EAEM algorithm, thus

\[
\frac{\partial P(k_i)}{\partial t} \approx m \prod_i f(E, k_i)k_i \sum_{j\text{-local-area}} f(E, k_j)k_j.
\]

In a network, the degrees \(k_i\) of most nodes are far smaller than their maximum \(k_{i(\text{max})}\), thus similarly we get

\[
\frac{\partial \prod_i}{\partial t} = \frac{m}{\sum_{j\text{-local-area}} f(E, k_j)k_j} f(E, k_i)k_i \sum_{j\text{-local-area}} f(E, k_j)k_j.
\]

Then, we get

\[
k_i = \frac{k_{i(\text{max})}}{k_{i(\text{max})} - m e^{\frac{k_i}{k_{i(\text{max})}^t}} + 1}.
\]

The probability density function of the degree of a node with remaining energy \(E\),

\[
P(k) = \int_{E_{\text{min}}}^{E_{\text{max}}} \rho P(E|k) dE = \int_{k_{i(\text{min})}}^{k_{i(\text{max})}} \frac{2LE_{\text{max}}}{m_0 + t E_{\text{max}}} \frac{1}{kk_{i(\text{max})}^t - k^t}.
\]

To obtain the overall probability density function, we also take a weighted average of these functions with the weights being the probabilities of remaining energy \(E\). Then

\[
P(k) = \int_{E_{\text{min}}}^{E_{\text{max}}} \rho P(E|k) dE = \int_{E_{\text{min}}}^{E_{\text{max}}} \frac{2LE_{\text{max}}}{m_0 + t E_{\text{max}}} \frac{1}{kk_{i(\text{max})}^t - k^t} dE,
\]

where \(\rho\) is the distribution of \(E\). \(k\) is the continuous random variable for node degree, \(E_{\text{min}}\) and \(E_{\text{max}}\) are the bounds of remaining energy.

In this model, \(P(k)\) is very versatile and it varies greatly with different energy distributions. Although the network structure may not be scale-free, it can balance the energy consumption of the whole network better than the previous algorithm. In the extreme case when \(k_{\text{max}} \to \infty\), the constraint of the links is canceled, then the EBEM algorithm will be equal to EAEM algorithm.

### 4. Numerical experiments

In the evolution of the network, we assume that the network starts with \(m_0 = 5\) nodes. At every step, a new node is connected to \(m = 3\) existing nodes. In the two evolution algorithms, we assume that the remaining energy of each node is varied from 0.5 to 1.0 \((0.5 \leq E \leq 1.0)\). We examine three different distributions of remaining energy in our experiments. The distribution functions and corresponding expected values are listed in Table 2.

In order to examine the performance of the networks constructed by EAEM, in the following, we will analyze the time evolution of the connectivity and the degree distribution. From Figs. 1–3, we can see how the factors (energy distribution, local-area and node incoming time) influence the connectivity growth of the node. Fig. 1 shows the impact of energy distribution on the connectivity growth. When the degree distribution \(\rho\) is given, the various expected value \(E\) will produce dif-

**Table 2**
The expected value for different distribution of remaining energy

<table>
<thead>
<tr>
<th>The distribution of remaining energy</th>
<th>(\rho(E)) distribution</th>
<th>The expected value of remaining energy (E)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform distribution, (\rho(E) = 2)</td>
<td></td>
<td>0.75</td>
</tr>
<tr>
<td>Power-law distribution, (\rho(E) \sim E^{-1})</td>
<td></td>
<td>0.5</td>
</tr>
<tr>
<td>Power-law distribution, (\rho(E) \sim E^{-3})</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>


different speed of connectivity growth. Fig. 2 predicts that the nodes newly introduced into the network will choose different numbers of nodes based on the size of local-area. Obviously, when \( L \) is small, the node \( i \) will have more probability to be linked than that of larger \( L \). Therefore, the connectivity \( k_i(t_i) \) will increase relatively fast when \( L \) is small. Fig. 3 shows the impact of \( t_i \), which is the time of a node introduced into the network on the connectivity growth. The smaller \( t_i \) is, the older the nodes will be. As the elder nodes have more time of being chosen in connecting the new coming nodes, the connectivity \( k_i(t_i) \) will increase more quickly than the younger ones. Fig. 4 shows the degree distribution of the networks generated by
EAEM. We find that the degree distribution has the power-law form despite of the distribution of remaining energy. It shows that we can get scale-free networks in any condition by using this algorithm. Therefore, we do not need to consider the form of remaining energy seriously while generating networks based on EAEM algorithm, and the produced networks always have a good performance of random error tolerance in spite of the various remaining energy of the sensor nodes.

In the following, we will analyze the features of networks generated by EBEM which may be different from the EAEM. From Eq. (17), we find that the degree distribution is very versatile for different energy distribution, and it is hard to get
universal final result. To present the time evolution for the connectivity and degree distribution, we study the case when the distribution of energy is uniform. Assume the nodes with the maximum energy $E_{\text{max}}$ can connect to $k_{\text{max}} = 20$ nodes at most.

Fig. 5 shows that the degree distribution of the networks constructed by EBEM algorithm when the remaining energy distribution is uniform. And the nodes in the networks with large degree will increase their degree slowly because of the constraint of the maximum degree. The algorithm prevents the nodes to consume their power unexpectedly, therefore the energy of the whole network will be consumed balanced. Obviously the EBEM algorithm can be more tolerant against attack error than EAEM algorithm, since there are many nodes with relatively large degrees in the network. Fig. 6 shows the time evolution for the connectivity in comparison with different size of local-area. Fig. 7 shows the time evolution for the connectivity influenced by different time of a node introduced to the network. From Figs. 6 and 7, we can see the speed decrease of the connectivity because of the maximum degree limitation.

5. Conclusion

In this paper, we give two methods for modelling WSNs. The first proposed model (EAEM) can organize the networks in an energy-efficient way, and this model can produce scale-free networks which can improve the networks reliance against random failure of the sensor nodes. In the second model (EBEM), the maximum number of links for each node is introduced into the algorithm. This algorithm can make energy consumption more balanced than the previous model (EAEM). Finally, we give the numerical experiments about the two models. It is believed that the two models provide some useful guidelines for different applications of WSNs.

Acknowledgements

This work is supported by the Foundation for Western Returned Chinese Scholars of the Ministry of Education, the National Natural Science Foundation of China (Grant No. 60673098) and the National Natural Science Foundation of China and the Research Grants Council of Hong Kong Joint Research Scheme (Grant No. 60731160826).

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