A simple coverage-evaluating approach for wireless sensor networks with arbitrary sensing areas

GaoJun Fan *, ShiYao Jin

National Laboratory for Parallel and Distributed Processing, National University of Defense Technology, ChangSha 410073, Hunan Province, PR China

Received 1 May 2007; received in revised form 4 November 2007; accepted 9 November 2007

Available online 17 November 2007
Communicated by L. Boasson

Abstract

We present a simple and fast deterministic solution to the area coverage problem in wireless sensor networks. The task is to determine whether all points in a region are covered by a given set of sensors, where each sensor may have any arbitrary sensing shape. We transform the area coverage problem to the intersection points’ coverage problem, which is simpler and more suitable for evaluating the area coverage problem than previous approaches in our view.

Keywords: Sensor network; Computational geometry; \( k \)-coverage; Arbitrary sensing areas

1. Introduction and related work

One fundamental issue in application of wireless sensor networks is to provide proper coverage of their deployment regions, which answers the questions about the quality of service (surveillance or monitoring) that can be provided by a sensor network with sensing or observing capability. Generally, the sensing area is considered as a perfect disk, where each sensor has heterogeneous or homogeneous sensing capability. As an example, we assume each sensor has a sensing radius of \( r_s \), and a point is covered by sensor \( s_i \) if and only if it lies within the disk of radius \( r_s \) centered at \( s_i \). Given a set of sensors deployed in a region \( R \) to be sensed, the coverage problem then becomes a matter of determining whether every point in \( R \) is covered by at least \( k \) sensors (namely, \( k \)-coverage problem).

There are many researches about coverage problem in regard to different objectives and metrics, but an aspect concerning coverage performance presented here dose not seem to appear explicitly in the literature. There are three previous works closely related to it. A solution based on geometric analysis for covering a convex region by using the same radius of disks is presented by Wang et al. in [1]. Huang and Tseng [2] solved the area coverage problem of the different and/or the same radius of disks by considering the perimeter coverage of disk. In recent works, a coverage evaluation criterion for covering any shape of monitored region by using any covering radii circle was given by Gallais [3]. We are not aware of any work that considers the problem of determining the coverage degrees of a...
region for wireless sensor networks with arbitrary sensing areas. The next section presents a simple and fast deterministic solution, which we feel is more suitable for evaluating the area coverage problem than previous approaches.

2. Problem and solution

We are given a set of sensing areas, \( A = \{a_1, a_2, \ldots, a_n\} \), and a covered region \( R \). Each area \( a_i, i = 1, \ldots, n \), may have any arbitrary shape and boundary \( B_i \). It can cover any point inside \( B_i \) (we say that a point is covered by \( a_i \) if the point lies in the interior of \( a_i \) but lies on \( B_i \) or lies in the exterior of \( a_i \)). Our task is to determine whether all points in \( R \) are covered by \( A \), namely, any point in \( R \) is covered at least one area in \( A \).

Intuitively, the area coverage problem can be solved by finding out all sub-regions divided by the coverage boundaries of all given areas (e.g., \( a_1, a_2 \) and \( a_3 \)), and then check if \( R \) is covered or not by all sub-regions. This is a difficult and computationally expensive job in geometry. Also, it may be difficult to calculate these sub-regions [4].

Instead of analyzing the area coverage problem, we try to look at how the intersection of any two boundaries of areas in \( A \) and/or the boundary of \( R \) is covered. For convenience, we define the set of intersection (SI) to be collection of points inside \( R \), which includes: (1) the intersecting points or the two end points of intersecting lines of any two boundaries of areas in \( A \); or (2) the intersecting points or the two end points of intersecting lines between any boundary of area in \( A \) and the boundary of \( R \).

On some occasions, we need to consider \( k \)-coverage problem. For example, in wireless sensor networks, because energy depletion, harsh environmental conditions, and malicious attacks may result in node failures or become inoperative at any time, it is desirable to have higher degrees of coverage. Here, we study the following \( k \)-coverage problem:

**k-coverage problem.** Given any integer \( k \), a monitored region \( R \) (or a curve \( L \)) is \( k \)-coverage by a set of sensing areas \( A \), where each area may have any arbitrary shape, if and only if each point in \( R \) (or on \( L \)) is covered by at least \( k \) areas in \( A \).

This problem is equivalent to determining whether each sub-region (sub-curve) divided by areas in \( A \) is covered by at least \( k \) areas. The following lemma shows that \( k \)-coverage problem of curve can be mapped to intersection point’ \( k \)-coverage problem, which can be efficiently solved.

**Lemma 1.** A curve \( L \) is \( k \)-coverage by a set of areas \( A = \{a_1, a_2, \ldots, a_n\} \), where each area may have any arbitrary shape if (1) each intersection point between any boundary of area in \( A \) and \( L \) is \( k \)-coverage; and (2) at least one point on \( L \) is \( k \)-coverage.

**Proof.** We prove by contradiction. Assume that the conditions (1) and (2) are satisfied, but there is a point \( P \) is covered by at most \((k - 1)\) areas in \( A \). Find the closest intersection point \( P' \) from \( P \) along \( L \). It is easy to see that \( P' \) is not \( k \)-coverage, which is a contradiction. □

We consider a special situation where there is no intersection point between \( L \) and boundaries. Obvious, if there is one point on \( L \) is \( k \)-coverage in this case, and then all points on \( L \) are \( k \)-coverage. Otherwise, there is at least one intersection point between \( L \) and boundaries. Next, we prove the theorem on \( k \)-coverage problem of region \( R \). The following result can allows us to transform the problem of determining the coverage degrees of a region to the simpler and more suitable problem of determining the coverage degrees of the intersection points.

**Theorem 1.** A region \( R \) is \( k \)-coverage by a set of areas \( A = \{a_1, a_2, \ldots, a_n\} \), where each area may have an arbitrary shape if (1) each point in SI is \( k \)-coverage by areas of \( A \); and (2) at least one point on the boundary of \( R \) is \( k \)-coverage by areas of \( A \).

**Proof.** We prove by using reduction of absurdity. Suppose there is no intersection point in SI which is not \( k \)-coverage, and at least one point on the boundary of \( R \) is \( k \)-coverage. Let \( X \) is a sub-region that is still not \( k \)-coverage in \( R \) (namely, \( X \) is covered by at most \((k - 1)\) areas in \( A \)). Furthermore, suppose all sub-regions around \( X \) are at least \( k \)-coverage. The boundary of \( X \) is collection of curves belong the boundaries of some areas in \( A \) and/or the boundary of \( R \), and it’s degrees of coverage is less than \( k \). Now we consider the following two cases:

1. The boundary of \( X \) includes curves belong at least two areas’ boundaries (as Fig. 1 illustrate, and gray sub-region denotes \( k \)-coverage). The intersection point(s) of curves (e.g., \( P_1, P_2, P_3 \) and \( P_4 \)) which belongs the boundaries of different areas are not \( k \)-coverage, which is a contradiction with assumption that all intersection points are at least \( k \)-coverage.
Fig. 1. A coverage patch bounded by at least two areas’ boundaries.

(2) The boundary of $X$ includes curves belong one area’ boundaries and the boundary of $R$. Because at least one point on the boundary of $R$ is $k$-coverage, the boundary of $R$ is $k$-coverage by $A$. Namely, the boundary of $X$ includes curves belong at least two area’ boundaries. Similarly as case (1), it contradicts with assumption that all intersection points are at least $k$-coverage.

From the above discussion, we can say that the region $R$ is $k$-coverage by $A$.  

We should note that the points on the boundary of sub-region $r_i$ have the same degrees with the points inside $r_i$. This is because a sub-region in $R$ is a set of points which are covered by the same set of areas.

3. Conclusion

We studied the problem of determining the coverage degrees of a region. By mapping the original problem to the problem of determining the coverage degrees of the intersection points, we can simply and fast evaluate the coverage performance when sensors with any arbitrary sensing areas are deployed in a region. Compared to the previous work that derives analytical coverage performance, our approach allows us to consider a wireless sensor network where (1) the covering region can be any region; (2) each sensor may have any arbitrary sensing shape.

References